Technical Notes

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Loss Factor for a Simply Supported Rectangular Plate of Variable Thickness

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I. Introduction

THE material damping of structural members subjected to multiaxial stress system plays an important role in the control of its vibratory response. This is of importance in the acoustical field, since the ratio of the internal loss factor to the radiation efficiency plays a vital role in the control of sound radiation from such members as a simply supported plate.[1] Lazan in his monograph[2] has reviewed the importance and discussed the evaluation of the internal loss factors of various members. Leissa[3] has reviewed almost all the investigations done previously to 1965 on plate vibrations. A large amount of work has been done on vibration of elastic plates of uniform thickness, but it appears that little work has been done on vibrations of rectangular plates of variable thickness, though such cases are of interest in the aeronautical field since they approximate wing sections. Appl and Byers[4] have studied the free vibrations of a simply supported rectangular plate having a linear thickness variation in one direction. Practically, no work is available on the analytical evaluation of modal damping of plates of variable thickness.

In the present Note, the classical thin-plate theory has been employed with complex rigidity to account for the energy dissipation. Approximate solutions of the plate equation, based on one-term and two-term normal mode solutions have been outlined by using Galerkin's method. The fundamental mode loss factors for the simply supported rectangular plate subjected to a central point harmonic load of single frequency, for different combinations of aspect ratios and linear taper parameters, have been evaluated. An approximate relationship has been obtained, correlating the loss factor for the plate of variable thickness with that of a plate of uniform thickness.

II. Forced Vibration of the Plate

The differential equation of motion of a rectangular plate with thickness variation in the x-direction is given by Appl and Byers.[4] Damping is taken into account by considering rigidity to be of complex form. Let the variable thickness be given by $h=h_0G(x)$ where $G(x)$, the taper function, is chosen as $[1+\alpha x/a]$. Here α represents the linear taper parameter. Considering the central point excitation p to be $(4P/ab)\sin(\pi x/a)\sin(\pi y/b)$, where P is the force amplitude and a and b are the length and width of the plate, one gets the approximate one-term and two-term response solutions as

$$W=A_{11}\sin(\pi x/a)\sin(\pi y/b) \tag{1}$$

and

$$W'=[A'_{11}\sin(\pi x/a)+A'_{21}\sin(2\pi x/a)]\sin(\pi y/b) \tag{2}$$

Applying Galerkin's method,[5] one obtains the coefficient A_{11} as

$$A_{11}=P/[(1+i\eta)B_0C_1-(\rho/g)h_0\omega_{11}^2C_2] \tag{3}$$

where η, B_0, ρ and ω_{11} are the loss factor, rigidity at $h=h_0$, density and fundamental frequency, respectively. The coefficients C_1 and C_2 are obtained as

$$C_1=\{\frac{\pi^4}{4a^2k}(1+k^2)^2[1+\frac{3\alpha}{2}+\alpha^2(1-\frac{3}{2\pi^2}) + \frac{\alpha^3}{4}(1-\frac{3}{\pi^2})]+1.5\frac{\pi^2k}{a^2}(1-\nu)\alpha^2(1+\frac{\alpha}{2})\}$$

$$C_2=(ab/4)[1+(\alpha/2)] \tag{4}$$

where $k=a/b$ is the aspect ratio.

Defining the frequency parameter λ as $\rho h_0a^4\omega_{11}^2/B_0g$ and equating the real part of the denominator of the coefficient A_{11} to zero in order to bring the phase of the resonant response in conformity to known behavior, one gets the frequency equation as

$$\lambda=C_1a^4/C_2 \tag{5}$$

and response equation

$$W=\frac{P\sin(\pi x/a)\sin(\pi y/b)}{i\eta B_0C_1} \tag{6}$$

The coefficients A'_{11} and A'_{21} of Eq. (2) can be obtained once again by following Galerkin's procedure and are obtained as

$$A'_{11}=\frac{P(F_2+i\eta A_2)}{B_0[F_1F_2-F_3F_4+\eta^2(A_1B_3-B_1A_2)+i\eta F_6]}$$

$$A'_{21}=\frac{-P(F_3+i\eta A_1)}{B_0[F_1F_2-F_3F_4+\eta^2(A_1B_3-B_1A_2)+i\eta F_6]}$$

It was found that A_2 and F_2 are of the same order. Therefore, for low damping, the imaginary term in the numerator of A'_{11} would contribute a very small phase and hence can be dropped. Now, in order to bring the phase of resonant response in conformity to known behavior, one obtains the frequency equation by equating the real part of the denominator of A'_{11} to zero (after neglecting η^2 term which is very small), as

$$F_1F_2-F_3F_4=0 \tag{7}$$

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The coefficient A'_{11} therefore reduces to

$$A'_{11} = (P/iB_0\eta)\cdot(F_2/F_6)$$

Similarly,

$$A'_{21} = -(P/iB_0\eta)\cdot(F_3/F_6)$$

where

$$F_1=(B_1+G_6),\ F_2=(A_2+H_6),\ F_3=(A_1+H_8)$$

$$F_4=(B_3+G_8)$$

$$F_6=(F_1A_2+B_1F_2)-(A_1F_4+B_3F_3)$$

$$A_1=(H_1+H_3+H_7),\ A_2=(H_2+H_4+H_5)$$

$$B_1=(G_1+G_3+G_5),\ B_3=(G_2+G_4+G_7)$$

$$G_1=(1+k^2)^2\frac{\pi^4}{4a^2k}[1+1.5\alpha$$

$$+\alpha^2(1-\frac{1.5}{\pi^2})+\frac{\alpha^3}{4}(1-\frac{3}{\pi^2})]$$

$$G_2=(2^2+k^2)^2\frac{\pi^2}{4a^2k}[\frac{\alpha^3}{3}(\frac{320}{9\pi^2}-8)$$

$$-\frac{16\alpha}{3}(1+\alpha)]$$

$$G_3=(1+k^2)\frac{\pi^2}{4a^2k}6\alpha^2(1+\alpha/2)$$

$$G_4=(2^2+k^2)\frac{\pi^2}{4a^2k}\cdot 8\alpha[2(1+\alpha)$$

$$-\alpha^2(\frac{52}{9\pi^2}-1)]$$

$$G_5=-(1+\nu k^2)\frac{\pi^2}{4a^2k}6\alpha^2(1+\alpha/2)$$

$$G_6=H_6=-\frac{\lambda}{4a^2k}(1+\alpha/2)$$

$$G_7=(2^2+k^2\nu)\frac{1}{4a^2k}\cdot\frac{32}{3}\alpha^3$$

$$G_8=H_8=\frac{\lambda}{4a^2k}\cdot\frac{16\alpha}{9\pi^2}$$

$$H_1=(1+k^2)^2\frac{\pi^2}{4a^2k}[\frac{\alpha^3}{3}(\frac{320}{9\pi^2}-8)$$

$$-\frac{16\alpha}{3}(1+\alpha)]$$

$$H_2=(2^2+k^2)^2\frac{\pi^4}{4a^2k}[1+1.5\alpha+\alpha^2$$

$$\times(1-\frac{1.5}{4\pi^2})+\frac{\alpha^3}{4}(1-\frac{3}{4\pi^2})$$

$$H_3=-(1+k^2)\frac{\pi^2}{4a^2k}\cdot 8\alpha[2(1+\alpha)+\alpha^2(1-\frac{28}{9\pi^2})]$$

$$H_4=(2^2+k^2)\frac{\pi^2}{4a^2k}\cdot 6\alpha^2(1+\alpha/2)$$

$$H_5=-(2^2+\nu k^2)\frac{\pi^2}{4a^2k}\cdot 6\alpha^2(1+\alpha/2)$$

$$H_7=(1+\nu k^2)\frac{1}{4a^2k}\cdot\frac{32}{3}\alpha^3$$

Thus, the two-term response can be obtained as

$$W'=\frac{P}{i\eta B_0F_6}\{F_2\sin\frac{\pi x}{a}-F_3\sin\frac{2\pi x}{a}\}\sin\frac{\pi y}{b} \tag{8}$$

$1/i$ in Eqs. (6) and (8) indicates the resonant phase of the response and can be dropped in the further analysis.

III. Loss Factor for the Plate

Having obtained the displacement, the normal stresses σ_x, σ_y, and τ_{xy}, and, thereafter, the principal stress σ_{a_1} and σ_{a_2} can be obtained at any point on the plate. The plate is now conceptually thought to consist of large number of small elements. Based on the modified theory of dilatational energy criterion,[2] which gives an upper bound of the loss factor value, an equivalent stress σ_e can be obtained at the center of each element. The loss factor can then be obtained from

$$\eta=(\frac{JE}{\pi})\{\left[\sum\sigma_e^N\right]/\left[\sum\sigma_e^2\right]\} \tag{9}$$

where the summation is carried through the elements. Here J and N are the damping constants of the material and the equivalent stress can be obtained as

$$\sigma_e=\sigma_{a_1}(1-2\nu\xi+\xi^2)^{1/N}(1+\xi)^{(N-2)/N} \tag{10}$$

where

$$\xi=[\sigma_{a_2}/\sigma_{a_1}]<1.0$$

IV. Approximate Relation Between Loss Factors of Uniform and Variable Thickness Plates

It is seen from Eqs. (6) and (10) that σ_{am} the maximum value of stress σ_e would be given by

$$\eta\sigma_{am}=\frac{0.75P\pi^2(1+\alpha/2)[(1+k^2\nu)\text{ or }(\nu+k^2)]}{h_0^2a^2C_1}$$

$$\times(1-2\nu\xi+\xi^2)^{1/N}(1+\xi)^{(N-2)/N}$$

Let $\phi=(\eta\sigma_{am})_V/(\eta\sigma_{am})_C$ where suffixes V and C represent variable and constant thickness plates. The latter corresponds to a thickness of h_0 when $\alpha=0$. Also, one gets the loss factor to be given by

$$\eta\alpha(\sigma_{am})^{N-2}\text{ or }\eta^{N-1}\alpha(\eta\sigma_{am})^{N-2}$$

where the proportionality constant includes the ratio of normalized damping and strain energy integrals.[2] The variation in this ratio was found to be negligible and, therefore, one obtains the approximate equation

$$\{\eta_V/\eta_C\}=\phi^{(N-2)/(N-1)} \tag{11}$$

where

$$\phi=[(1+\alpha/2)\cdot C_{1_C}/C_{1_V}$$

Table 1 Frequency parameter λ

S.No.	k	α	λ mean (Ref.4)	One-term approx. [Eq. (5)]	% Error	Two-term approx. [Eq. (7)]	% Error
		0.1	167.657	168.053	0.236	167.660	0.0017
		0.2	183.581	185.173	0.867	183.600	0.0103
		0.3	199.972	203.560	1.794	200.037	0.0327
		0.4	217.082	223.213	2.824	216.990	−0.0424
1	0.5	0.5	234.215	244.133	4.235	234.480	0.1132
		0.6	251.944	266.320	5.706	252.528	0.2317
		0.7	270.139	289.773	7.268	271.152	0.3752
		0.8	288.786	314.492	8.901	290.373	0.5495
		0.1	429.344	430.366	0.238	429.355	0.0026
		0.2	470.539	474.641	0.872	470.601	0.0133
		0.3	513.154	522.456	1.813	513.413	0.0505
		0.4	557.195	573.813	2.982	557.850	0.1175
2	1.0	0.5	603.011	628.711	4.262	603.969	0.1589
		0.6	650.051	687.150	5.707	651.829	0.2735
		0.7	698.483	749.129	7.251	701.485	0.4298
		0.8	748.214	814.651	8.879	752.988	0.6381

Table 2 Loss factor ratio η_V/η_C

S. No.	k	α	η_V/η_C [Eq. (11)]	η_V/η_C (one-term approx.)	η_V/η_C (two-term approx.)
		0.1	0.9782	0.9786	0.9842
		0.2	0.9573	0.9579	0.9701
		0.3	0.9374	0.9387	0.9566
		0.4	0.9184	0.9202	0.9441
1	0.5	0.5	0.9002	0.9026	0.9326
		0.6	0.8830	0.8859	0.9212
		0.7	0.8666	0.8702	0.9107
		0.8	0.8509	0.8553	0.9005
		0.1	0.9781	0.9783	0.9840
		0.2	0.9571	0.9576	0.9696
		0.3	0.9368	0.9383	0.9563
		0.4	0.9175	0.9198	0.9439
2	1.0	0.5	0.8991	0.9022	0.9322
		0.6	0.8815	0.8855	0.9208
		0.7	0.8647	0.8698	0.9101
		0.8	0.8487	0.8545	0.8997

V. Conclusions

For the purpose of analysis, a steel plate of SAE 1020 was considered, having a thickness of $h_0 = 0.1$ in. (0.0025m) and damping properties[2] J and N as 2.626×10^{-13} and 2.286, respectively, and subjected to a central point load of 0.2248 lb (1*N*). Fundamental mode loss factors were computed for the various combinations of the aspect ratios and taper parameters.

Table 1 shows the values of the frequency parameters for two representative cases. It is observed that the two-term solution gives a fairly accurate value of the natural frequency, and the loss factors calculated at these values of the resonant frequencies would not be far-off from the actual values. Table 2 shows the values of the loss factors as calculated by one-term and two-term approximations, and as obtained from Eq. (11). It is seen that Eq. (11) gives a value of loss factor which is within a maximum of 5% of the two-term approximation. Hence, this could be used for obtaining an estimate of the fundamental mode internal loss factor of the plate.

References

[1] Ver, I. L. and Holmer, C. I., "Interaction of Sound Waves with Solid Structures," *Vibration and Noise Control*, edited by L. L. Beranek, 1st ed., McGraw-Hill, New York, 1971, pp. 331-333.

[2] Lazan, B. J., *Damping of Materials and Members in Structural Mechanics*, 1st ed., Oxford-Pergamon, New York, 1968.

[3] Leissa, A. W., *Vibration of Plates*, SP-160, 1969, NASA.

[4] Appl, F. C. and Byers, N. R., "Fundamental Frequency of Simply Supported Rectangular Plates of Linear Varying Thickness," *Journal of Applied Mechanics*, Vol. 32, 1965, pp. 163-167.

[5] Kantorovich, L. V. and Krylov, V. I., *Approximate Methods of Higher Analysis*, 3rd ed., Interscience, New York, 1964, pp. 258-262.

Hypersonic Flow over Concave Surfaces with Leading-Edge Bluntness

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Introduction

THE study of leading-edge bluntness effects at hypersonic speeds on surfaces other than a flat plate or a convex afterbody shape has not received much attention in the literature. The nature of the downstream effect of the leading-

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